

A TRANSIENT VIBRATION INTENSITY ANALYSIS - Application of Envelope Intensity -

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ABSTRACT

The envelope intensity method, which is useful for investigating transient behavior of the vibration, is presented. In most cases, the vibration intensity is time averaged and, therefore, it is basically applicable only to stationary vibrations. However, it is important to investigate transient behavior of the vibration and its energy flow. Using the time averaged intensity in those cases, we miss the information on time dependent property. On the other hand, the instantaneous intensity changes too fast in time, making the observation of the phenomenon difficult. The envelope intensity was proposed in order to keep the time dependent property which varies slowly as the envelope of the original signal does. In this paper the envelope intensity applied to the vibration intensity analysis is reported.

KEY WORDS : vibration intensity, transient vibration, envelope intensity

1. INTRODUCTION

The intensity analysis technique which can make the energy flow visible, is applied to the vibration intensity. The time averaged intensity is used in most cases. Since this method needs only the cross-spectrum of two sensor output, it is easy to measure the intensity by using a 2-channel FFT analyzer. However, it is very important to investigate the transient phenomenon, when we search for the sources and the transmission paths of the vibration. The time averaged intensity not only loses a peak value and time dependent information, but also changes its value by the length of averaging time. On the other hand, instantaneous intensity changes its value too fast, making it too difficult to observe the effective energy flow. In this paper, we introduce an envelope intensity and compare it with the instantaneous intensity [1].

2. DEFINITION

The instantaneous vibration intensity $I(t)$ of a bending wave is given by [2]

$$I(t) = Q(t)\xi'(t) + M(t)\theta_m'(t) + T(t)\theta_r'(t) \quad (1)$$

where $Q(t)$ is the shear force, $M(t)$ is the bending moment, $T(t)$ is the torsional moment, $\xi(t)$ is displacement, $\theta_m(t)$ and $\theta_r(t)$ are angular displacements, and ' denotes the differential operation. Eq.(1) can be written in the following form:

$$I(t) = [F][V] \quad (2)$$

where

$$[F] = [Q(t) \ M(t) \ N(t)] \quad (3)$$

$$[V]^T = [\xi'(t) \ \theta_m'(t) \ \theta_r'(t)] \quad (4)$$

In this way, the vibration intensity can be dealt with the same way as the sound intensity by replacing a vector of the force terms $[F]$ and transverse velocity terms $[V]$ by the sound pressure $p(t)$ and the particle velocity $v(t)$, respectively. Since the sound intensity is much simpler than the vibration intensity, the idea of the envelope intensity will be described by use of the sound intensity. The instantaneous sound intensity $i(t)$ is given by

$$i(t) = p(t)v(t) \quad (5)$$

where $p(t)$ and $v(t)$ are the sound pressure and the particle velocity, respectively. For example, considering the signals $p(t) = P\cos(2\pi ft)$ and $v(t) = V\cos(2\pi ft + \phi)$, $i(t)$ is given by

$$i(t) = PV[\cos\phi + \cos(4\pi ft)]/2 \quad (6)$$

The alternative component of the instantaneous intensity changes with the twice of the original frequency. This property of the instantaneous intensity makes the observation of the direction of the energy flow difficult. The time averaged intensity I is given by

$$I = \overline{i(t)} = \frac{1}{T} \int_{-T/2}^{T/2} i(t) dt \quad (7)$$

where $\overline{\quad}$ denotes the time average operation, T is the average time. By using the former signal, since they are periodic, I becomes

$$I = (PV\cos\phi)/2 \quad (8)$$

which is independent of time. Next, let's consider a rectangular pulse :

$$p(t) = P \quad (-a \leq t \leq a) \\ = 0 \quad (\text{otherwise})$$

$$v(t) = V \quad (-a \leq t \leq a) \\ = 0 \quad (\text{otherwise})$$

In this case $i(t)$ and I are given, respectively, by

$$i(t) = PV \quad (-a \leq t \leq a) \\ = 0 \quad (\text{otherwise})$$

$$I = PV \quad (T \leq a) \\ = aPV/T \quad (T > a)$$

Thus the time averaged intensity becomes the function of the averaging time T . When treating the transient signal, the time averaged intensity is not so useful, because it gives the result which depends on the length and the position of the observation interval.

The envelope intensity varies as the envelope of the original signal and shows the time dependent property of the intensity. There are several methods to obtain the envelope of a signal. A method which uses the Hilbert transform is useful in many cases. The analytic function has the original signal for its real part and Hilbert transform of the original signal for its imaginary part. The analytic functions of $p(t)$ and $v(t)$ are given, respectively, by

$$p_a(t) = \frac{1}{\sqrt{2}} \{ p(t) - jH[p(t)] \} \quad (9)$$

$$v_a(t) = \frac{1}{\sqrt{2}} \{ v(t) - jH[v(t)] \} \quad (10)$$

where $p_a(t)$ and $v_a(t)$ is the analytic function of $p(t)$ and $v(t)$ (assuming $p(t)$ and $v(t)$ are real), respectively, j is the imaginary unit, and $H[\cdot]$ denotes the Hilbert transform operation. The analytic function defined above has the same energy as the original function but only with positive frequency spectra. The envelope intensity is defined, by using the former analytic function, as follows :

$$I_e(t) = \text{Re}[p_a(t)v_a^*(t)] \quad (11)$$

where $Re[\cdot]$ and $*$ indicate the real part and the complex conjugate, respectively.

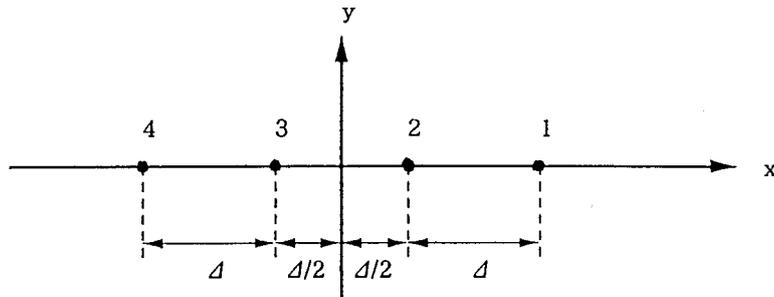


Fig.1 A four sensor array.

The same thing can be applied to vibration intensity analysis. The envelope intensity of the vibration intensity $I_s(t)$ is given by

$$I_s(t) = Re \left[[F_s] [V_s]^* \right] \\ = Re \left[Q_s(t) \xi_s^{**}(t) + M_s(t) \theta_{\mu s}^{**}(t) + T_s(t) \theta_{\tau s}^{**}(t) \right] \quad (12)$$

where the functions with suffix a indicate the analytic functions of the original time signals, respectively. Since there is no convenient method to obtain the angular displacements directly, the velocity terms in Eq.(1) are obtained approximately from the output of the sensors of a linear array as shown in Fig. 1. The array consists of 2 or 4 sensors with a Δ spacing. In case of the linear array, the torsional moment is neglected and a one-dimensional wave is assumed.

Eq.(1) becomes

$$I(t) = Q(t) \xi'(t) + M(t) \theta_{\mu}'(t) \quad (13)$$

and Eq.(2) becomes

$$I_s(t) = Re \left[Q_s(t) \xi_s^{**}(t) + M(t) \theta_s^{**}(t) \right] \quad (14)$$

$Q(t)$, $M(t)$, and $\theta_{\mu}(t)$ are related to the flexural displacement $\xi(t)$ as follows :

$$Q(t) = B \frac{\partial^3 \xi(t)}{\partial x^3} \quad (15) \quad M(t) = -B \frac{\partial^2 \xi(t)}{\partial x^2} \quad (16) \quad \theta_{\mu}(t) = -\frac{\partial \xi(t)}{\partial x} \quad (17)$$

where B is the bending stiffness, x is the direction of the sensor array. In case of the 4 sensor array, these are approximated as

$$Q(t) \approx \frac{B}{\Delta^3} [\xi_1(t) - 5\xi_2(t) + 5\xi_3(t) - \xi_4(t)] \quad (18)$$

$$M(t) \approx -\frac{B}{2\Delta^2} [\xi_1(t) - \xi_2(t) - \xi_3(t) + \xi_4(t)] \quad (19)$$

$$\theta_{\mu}(t) \approx \frac{1}{\Delta} [\xi_2(t) - \xi_3(t)] \quad (20)$$

where $\xi_i(t)$ denotes the displacement at the i th sensor in Fig.1. From the Fourier transforms of the above equations we obtain

$$Q(\omega) \approx \frac{B}{\Delta^3} [\xi_1(\omega) - 5\xi_2(\omega) + 5\xi_3(\omega) - \xi_4(\omega)] \quad (21)$$

$$M(\omega) \approx -\frac{B}{2\Delta^2} [\xi_1(\omega) - \xi_2(\omega) - \xi_3(\omega) + \xi_4(\omega)] \quad (22)$$

$$\theta_{\mu}(\omega) \approx \frac{1}{\Delta} [\xi_2(\omega) - \xi_3(\omega)] \quad (23)$$

The velocity spectra $\xi'(\omega)$ and $\theta_{\mu}'(\omega)$ are given as

$$\xi'(\omega) = \frac{j\omega}{2} [\xi_1(\omega) + \xi_2(\omega)] \quad (24)$$

$$\theta_{\mu}'(\omega) = \frac{j\omega}{\Delta} [\xi_1(\omega) - \xi_2(\omega)] \quad (25)$$

A procedure to obtain the analytic functions for Eqs.(21) - (25) are as follows :

- (1) Let the negative frequency components be zero.
- (2) Multiply the positive frequency components by $\sqrt{2}$.
- (3) Then apply the inverse Fourier transform.

Substitution of these results into Eq.(14) gives the envelope intensity.

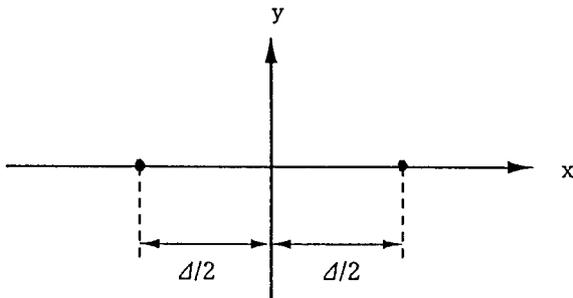


Fig.2 A two sensor array.

In case of using a 2 sensor array (Fig.2), the far field condition is assumed and, therefore, the following approximations can be used.

$$\frac{\partial^2 \xi(\omega)}{\partial x^2} \approx -K^2 \xi(\omega) \quad K^2 = \frac{m\omega^2}{B} \quad (26)$$

$$\xi(\omega) \approx \frac{1}{2} [\xi_1(\omega) + \xi_2(\omega)] \quad (27)$$

$$\frac{\partial \xi(\omega)}{\partial x} \approx \frac{1}{\Delta} [\xi_1(\omega) - \xi_2(\omega)] \quad (28)$$

where $\xi_i(\omega)$ denotes the displacement spectrum at i th sensor in Fig.2, m is the mass per unit length. Substituting Eqs.(26),(27),(28) into Eqs.(15),(16),(17) gives

$$Q(\omega) \approx -\frac{\sqrt{Bm}\omega}{\Delta} [\xi_1(\omega) - \xi_2(\omega)] \quad (29)$$

$$M(\omega) \approx \frac{\sqrt{Bm}\omega}{2} [\xi_1(\omega) + \xi_2(\omega)] \quad (30)$$

$$\xi'(\omega) \approx \frac{1}{j2\omega} [\xi_1(\omega) + \xi_2(\omega)] \quad (31)$$

$$\theta_{\mu}'(\omega) \approx \frac{1}{j\Delta\omega} [\xi_1(\omega) - \xi_2(\omega)] \quad (32)$$

The envelope intensity can be obtained by the same as in the 4 sensor case.

3.EXPERIMENT

An example of the application of the envelope intensity to a transient transverse vibration of a thin beam is shown here. The beam with dimensions of $1000 \times 30 \times 2$ (mm) and the excitation and sensor locations are shown in Fig.3. The beam is sandwiched by rubber sheets with a high damping. The beam was hit by an impulse hammer at the excitation point. The transverse vibrations at two closely located points were measured by two accelerometers. Fig.4 shows measurement results. Fig.4 (a) is one of the original acceleration signal. Fig.4 (b) and (c) show the instantaneous and envelope intensities, respectively.

From these two charts, it can be said that the instantaneous intensity changes its sign with higher frequencies than the original time signal and it does not give useful information for the understanding of the energy flow. The envelope intensity, on the other hand, shows clearly the forward and backward energy flows, making it much easier to understand the time - dependent energy flows. Fig.4 (d) shows a band - pass filtered envelope intensity with a center frequency at 1 KHz. The filtered envelope intensity shows more clearly the arrival times of the direct and reflected intensity flows.

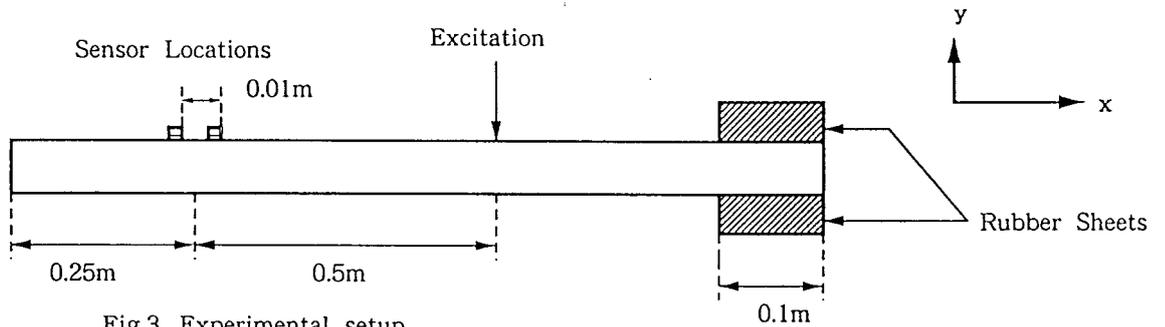


Fig.3 Experimental setup.

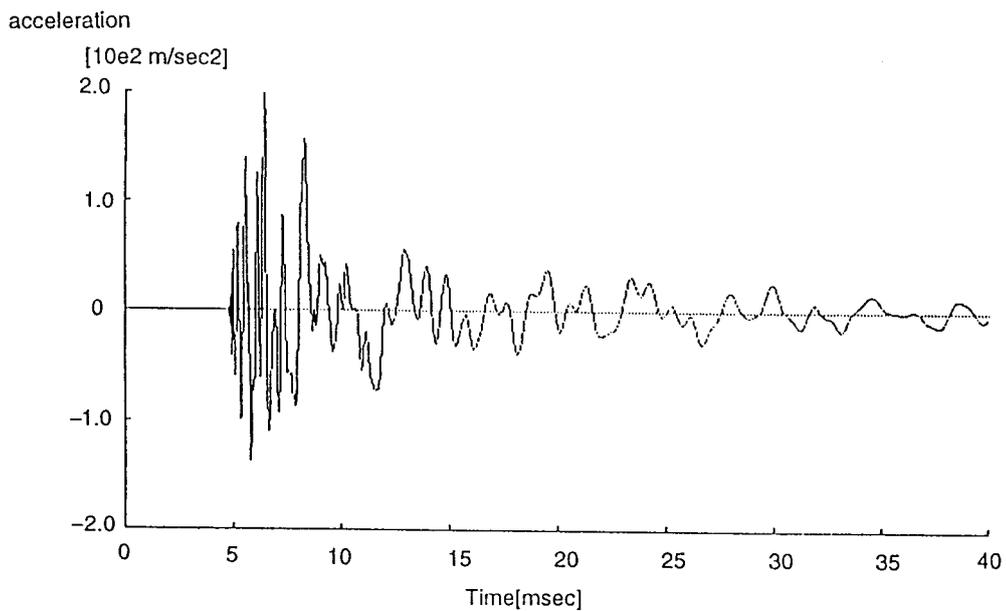


Fig4 (a) Acceleration signal.

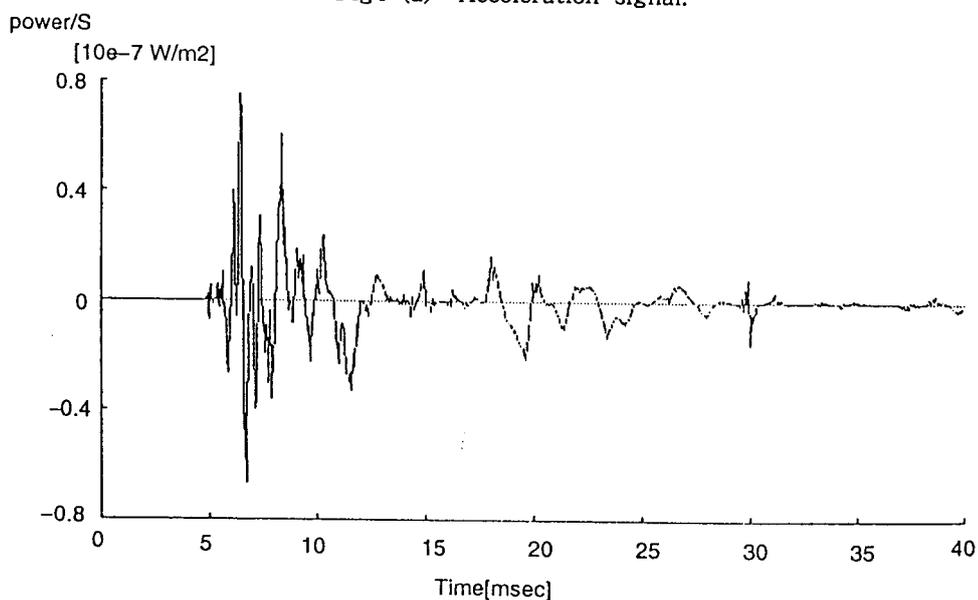


Fig4 (b) Instantaneous intensity.

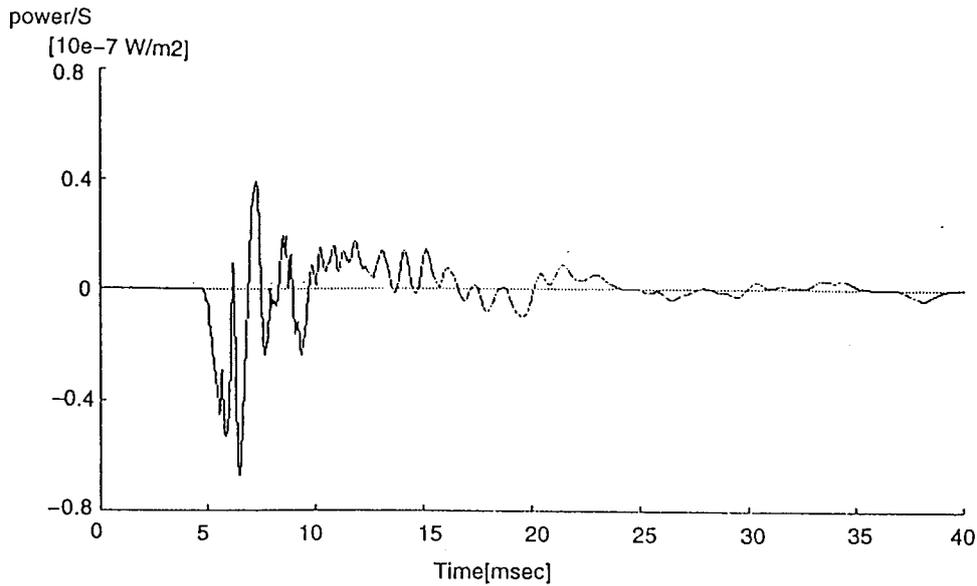


Fig.4 (c) Envelope intensity.

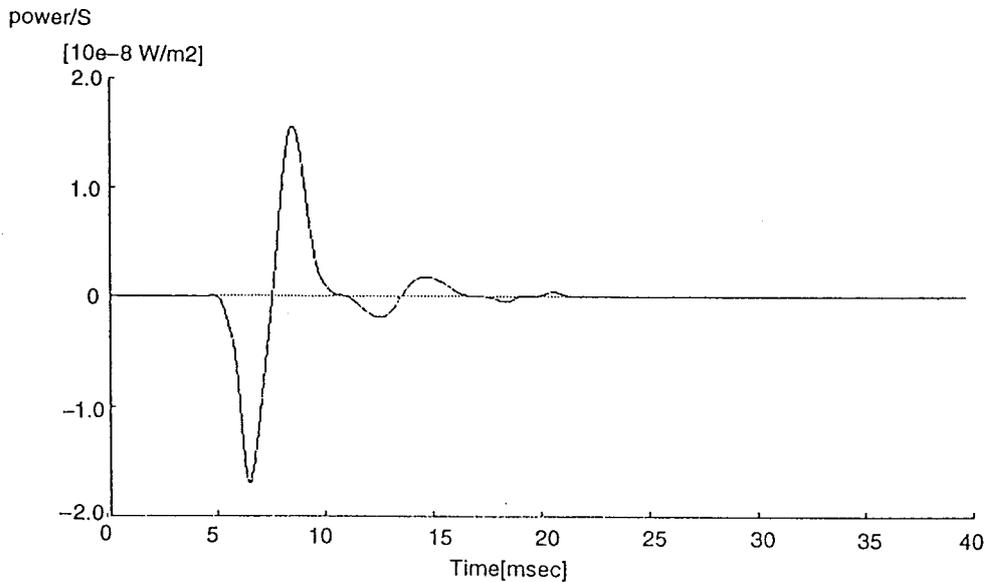


Fig.4 (d) Band - pass filtered envelope intensity.

4.CONCLUSIONS

An application of the envelope intensity to a vibration intensity was introduced. An example of the application clearly shows that the envelope vibration intensity has a great advantage in understanding the transient behaviour of structural vibrations. A usefulness of the envelope vibration intensity will be tested by applying it to real structures in the near future.

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